

EXERCISES [MAI 2.17]

SINUSOIDAL MODEL

SOLUTIONS

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A. Paper 1 questions (SHORT)

1.

Function	Amplitude	Period	Range
$f(x) = \sin x^\circ$ $f(x) = \cos x^\circ$	1	360	$-1 \leq y \leq 1$
$f(x) = \sin(10x)^\circ$ $f(x) = \cos(10x)^\circ$	1	36	$-1 \leq y \leq 1$
$f(x) = \sin(36x)^\circ$ $f(x) = \cos(36x)^\circ$	1	10	$-1 \leq y \leq 1$
$f(x) = \sin(180x)^\circ$ $f(x) = \cos(180x)^\circ$	1	2	$-1 \leq y \leq 1$
$f(x) = 2 \sin(10x)^\circ$ $f(x) = 2 \cos(10x)^\circ$	2	36	$-2 \leq y \leq 2$
$f(x) = \sin(10x)^\circ + 50$ $f(x) = \cos(10x)^\circ + 50$	1	36	$49 \leq y \leq 51$
$f(x) = 2 \sin(10x)^\circ + 50$ $f(x) = 2 \cos(10x)^\circ + 50$	2	36	$48 \leq y \leq 52$
$f(x) = -2 \sin(10x)^\circ$ $f(x) = -2 \cos(10x)^\circ$	2	36	$-2 \leq y \leq 2$
$f(x) = -2 \sin(10x)^\circ + 50$ $f(x) = -2 \cos(10x)^\circ + 50$	2	36	$48 \leq y \leq 52$
$f(x) = 100 - 20 \sin(10x)^\circ$ $f(x) = 100 - 20 \cos(10x)^\circ$	20	36	$80 \leq y \leq 120$

2. (a) central axis $y = 100$, amplitude = 80, period = 2

(b) $B = \frac{360}{\text{Period}} = \frac{360}{2} = 180$, It is of type +sin

$$f(x) = 60 \sin(180x)^\circ + 100,$$

(c) When $x = 1.5$, $y = 40$. When $x = 1.6$, $y = 42.9$.

(d) 0.232

3. (a) central axis $y = 50$, amplitude = 30, period = 180

(b) $B = \frac{360}{\text{Period}} = \frac{360}{180} = 2$, It is of type $-\sin$

$$f(x) = -30\sin(2x)^\circ + 50,$$

(c) $x = 20.9$, $x = 60.1$

(d) $20.9 \leq x \leq 60.1$

4. (a) central axis $y = 50$, amplitude = 30, period = 10

(b) $B = \frac{360}{\text{Period}} = \frac{360}{10} = 36$, It is of type $+\cos$

$$f(x) = 30\cos(36x)^\circ + 50,$$

(c) $x = 1.34$, $x = 8.66$, $x = 11.3$, $x = 18.7$

(d) $0 \leq x \leq 1.34$ or $8.66 \leq x \leq 11.3$ or $18.7 \leq x \leq 20$.

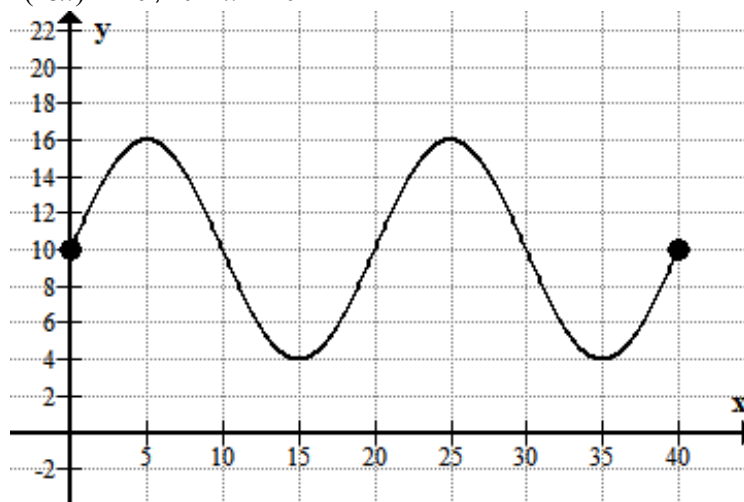
5. (a) central axis $y = 13$, amplitude = 5, period = 20

(b) $B = \frac{360}{\text{Period}} = \frac{360}{20} = 18$, It is of type $-\cos$

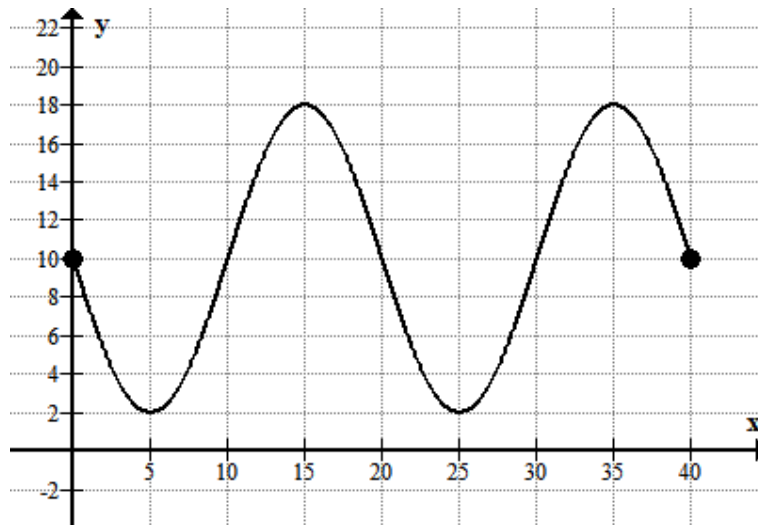
$$f(x) = -5\cos(18x)^\circ + 13.$$

(c) $x = 0$, $x = 20$, $x = 40$.

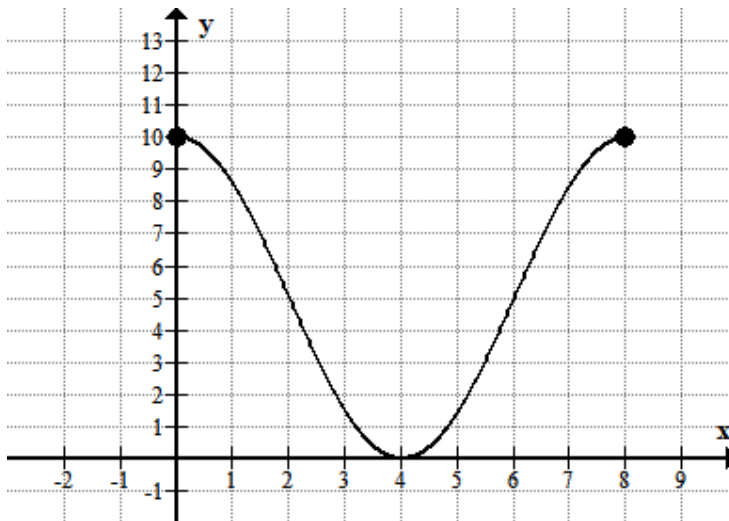
6. $f(x) = 6\sin(18x)^\circ + 10$, $0 \leq x \leq 40$



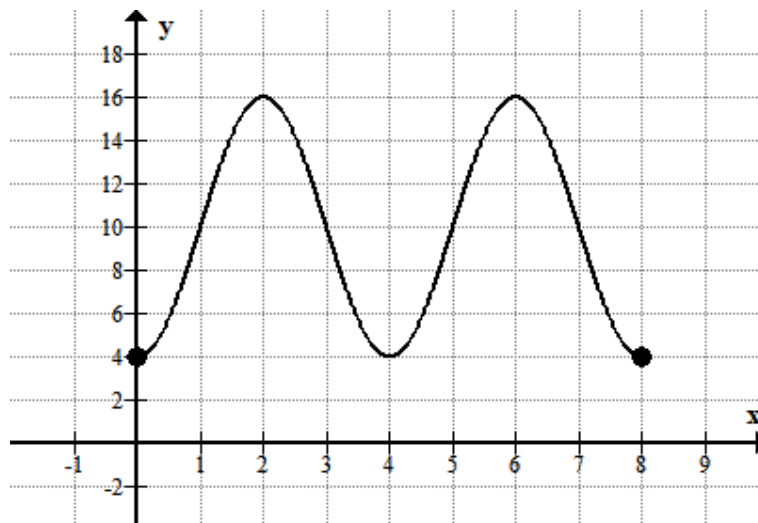
7. $f(x) = -8\sin(18x)^\circ + 10$, $0 \leq x \leq 40$



8. $f(x) = 5 \cos(45x)^\circ + 5, 0 \leq x \leq 8$

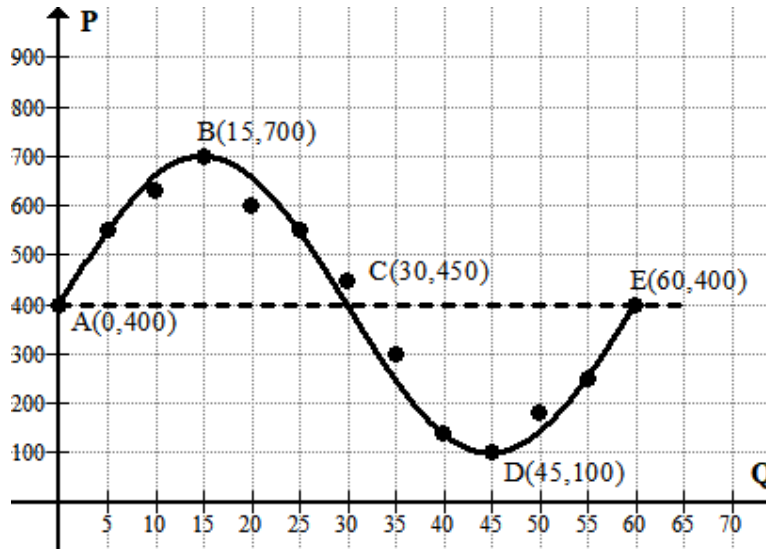


9. $f(x) = -6 \cos(90x)^\circ + 10, 0 \leq x \leq 8$



B. Paper 2 questions (LONG)

10. (a) (i) $P = 400$
 (ii) amplitude = 300
 (iii) Period = 60
 (b) $B = \frac{360}{\text{Period}} = \frac{360}{60} = 6$, It is of type +sin
 $P = 300 \cos(6Q)^\circ + 400$.
 (c)



- (d) When $Q = 30$, $P_A = 300 \sin(6 \times 30)^\circ + 400 = 400$, $P_E = 450$

$$\text{Percentage error} = \left| \frac{P_A - P_E}{P_E} \right| \times 100\% = \left| \frac{400 - 450}{450} \right| \times 100\% = 11.1\%$$

- (e) (i) When $Q = 90$,
 (ii) Extrapolation: We are not certain if the data continue periodically.

11. (a) When $t = 1$, $l = 33 + 5 \cos 720 = 38$
 (b) (i) $l_{\min} = 33 - 5 = 28$ (ii) $l_{\max} = 33 + 5 = 38$
 (c) $33 = 33 + 5 \cos 720t \Rightarrow t = 1/8$
 (d) period = $\frac{360}{720} \left(= \frac{1}{2} \right)$
 (e) (i) 10 times (ii) 20 times